With the Authors Compliments

A

NON-EUCLIDEAN THEORY OF MATTER AND ELECTRICITY

BY

P. A. CAMPBELL

FOR SALE BY

GEORGE H. KENT

University Bookstore, Cambridge, Mass.

PRICE, 35 CENTS

A NON-EUCLIDEAN THEORY OF MATTER AND ELECTRICITY

BY

P. A. CAMPBELL

FOR SALE BY

GEORGE H. KENT

University Bookstore, Cambridge, Mass.

PRICE, 35 CENTS

Published July, 1907

THE UNIVERSITY PRESS, CAMBRIDGE, U. S. A.

A NON-EUCLIDEAN THEORY OF MATTER AND ELECTRICITY

"I hold . . . that in the physical world nothing else takes place but this [spatial] variation." — W. K. CLIFFORD: *On the Space-Theory of Matter.*

I

THE theory of matter and electricity which I shall undertake to present in this essay is based first of all on the broad assumption that there exists a physical substratum — the so-called ether — which, with its states and mutations, wholly constitutes the physical universe. Other, less general assumptions of the theory are the following: —

1. The ether is self-extended; that is, what we are accustomed to call space is the general, constitutional extension of the ether.

2. The ether is self-existing; or, time is the continuous existence of the ether.

3. The ether is continuous, limitless, and indestructible.

4. Space, or the constitutional extension of the ether, contains both variety and change of form. In other words, nonhomogeneity and mutability are attributes belonging to space.

5. The normal form of space is the Euclidean form; or, every smallest deviation of space from the homogeneous, Euclidean form is accompanied by appropriate forces in the substance of the ether which tend to restore the spatial relations of its parts to the mathematical relations of Euclidean geometry.

6. Lastly, spatial distances and time intervals are mutually comparable, the relationship between the two being such that 186,000 miles of distance corresponds with, or in a sense equals, one second of time.

Beginning now on the general sketch of the theory, let us start, in accordance with postulate 3, with a continuous, limitless,

and indestructible universe of ether; and let us suppose that at first its extension is strictly Euclidean throughout. That is, let it be supposed that the quantity of ether is everywhere just sufficient so that its general extension, or space, has everywhere the properties of Euclidean geometry. Then this universe of ether is, by assumption 5, in a strictly normal condition and void of force. The result is a stereotyped continuum which contains neither physical energy nor physical manifestation of any kind. It thus resembles the actual universe only in possessing three-dimensional extension. Now, according to the present theory, the whole problem of introducing the entity matter into this lifeless cosmos is merely a question of bringing just the right distribution of additional ether into conjunction with the old; or, what amounts to the same thing conceptually, of amplifying, in a properly varied manner, this as yet undifferentiated substratum. To begin with, I shall attempt to discuss the state — essentially stable state — of the ether in the simplest possible case, viz., the case in which there subsists in the whole body of ether but a single, motionless unit of matter. By a unit of matter I here mean the entity constituted, according to the present theory, by a negative electron when it has "lost" its negative electrification. Loosely speaking, it is the entity defined by the phrase "an unelectrified negative electron."

It follows from the two attributes of continuity and indestructibility with which we endowed the ether that, upon the introduction, in any manner, of an additional quantity of the same, the resulting volume of the whole must be greater than the previous volume by just the quantity of the added ether. For the ether being continuous, its volume measures its quantity; and it being indestructible, its volume is necessarily constant.

Furthermore, since our universe is assumed limitless, there is no possibility that the added ether may annex itself to the old in a normal manner at a bounding surface of the latter. That is, the surplus ether must constitute, permanently, a real excess of volume.

It is seen, then, that the ether is a perfectly incompressible

continuum, in which the entire effect of an excess in quantity is an excess in volume. In other words, the ether is a substratum which is subject to spatial distortion only, and is wholly incapable of existing in a state of compression.

With this circumstance in mind, let us now assume that about some point of our normal universe of ether there occurs a general amplification of volume. And suppose this process takes place in accordance with the following geometrical scheme: Consider the series of all possible solid spheres of ether having as common centre the point in question. Then let the uniform, three-dimensional amplification of these spheres be inversely proportional to the sizes of their respective surfaces previous to amplification.

By a uniform, three-dimensional amplification of a solid sphere of ether I mean such a simultaneous growth of each small portion of the sphere that the resulting solid is also a sphere, every possible cross-section through the latter being but the magnified image of a corresponding cross-section through the former. Of course, in this process, it is supposed that the total quantity of ether in the universe is increased.

The above formula applies, as stated, to every conceivable solid sphere, having as its centre the fixed point about which the general amplification takes place. It is necessary, however, to suppose that no one of these possible spheres is amplified till the infinite number of still larger spheres have all in turn received their appropriate amplifications. Evidently, according to this scheme, any finite sphere consists, at the time of its own enlargement, not simply of a portion of the original ether, but a solid sphere of the original ether already amplified uniformly an infinite number of times. But now it is plain that uniform, three-dimensional amplification introduces no abnormal character whatever into the whole body of ether thus operated upon. Hence it follows that up to, and including, the time when each solid sphere is amplified, the ether lying wholly within the limits of that sphere continues in a strictly Euclidean condition. But there is an abnormal character introduced into the spatial relations of the ether at the

limits of the spheres. Consequently when the process has been carried up to the point-centre itself, an abnormal spatial condition has been imposed on the whole etheric continuum. At the centre the abnormal character is very pronounced, while away from it the abnormity is less, vanishing, however, only at an infinite distance therefrom. Thus the universe of ether now contains a spatial *inequality* of infinite extent. *This general extensional inequality constitutes, according to the present theory, a single elementary unit of matter.*

We have now considered the relative amplifications of the spheres only, and have still to consider their absolute amplifications. Now the absolute amplifications of the spheres are determined by the obviously necessary condition that the spheres, even after their respective growths, must become smaller and smaller without limit as the fixed centre is approached. This we proceed to show.

Let dS represent the increment in the surface area of a sphere upon its amplification, and S represent the surface area. Then the fraction dS/S will be our measure of the amplification for that whole sphere. Let also R be the radius of the sphere. Then $S=4\pi R^2$, and $dS=8\pi RdR$. Therefore the amplification is

$$\frac{dS}{S}=\frac{8\pi RdR}{4\pi R^2}=2\frac{dR}{R}.$$

That is, if it were required that dR be kept constant for the several spheres, their amplifications would have to be inversely proportional to their radii.

This means that if the relative amplifications of the spheres were inversely as their radii, instead of inversely as their surface areas — or inversely as the squares of the radii — and if the absolute amplifications of the spheres were such that in any part of their whole range the inside spheres were smaller than the outside, then throughout the whole range of spheres the inside spheres would be the smaller, and would grow smaller without limit as the centre was approached. For positive amplifications, such as we are considering, this rate of decrease in the size of the spheres

would, of course, be abnormally slow. Moreover it is seen that this abnormally slow rate of decrease is uniform throughout the whole range, so that as the degree of the absolute amplification is gradually increased, the limit at which the inside spheres cease to become smaller is reached simultaneously by all the spheres.

But now the actual amplifications of the spheres were made inversely proportional to their surface areas. In this case, unless the absolute amplifications be taken very small, a point will be reached, in approaching the centre, at which the inner spheres cease to grow smaller. It is evident that in such event the centre can be reached only by abandoning the general scheme of amplification.

To illustrate this, suppose that on this scheme the absolute amplifications are such that a sphere of radius R_1 is so greatly enlarged by its amplification, $2d_1R/R_1$, that the increment d_1R in its radius is the nth part of the absolute increment therein which would make the inner spheres cease at that point from becoming smaller. Then if we pass to a sphere of radius $R_2 = R_1/n$ it will be enlarged by the amount

$$2\frac{d_2R}{R_2} = 2n^2\frac{d_1R}{R_1} = 2\frac{nd_1R}{R_2}.$$

That is, in this case $d_2R = nd_1R$. Hence it follows that at this point the inner spheres cease to become smaller, and the scheme of amplification breaks down.

But the scheme must hold however near we approach the central point. It is essential, therefore, that every sphere of finite radius be amplified by an amount equal only to an infinitesimal of a higher order. For the amplification of such a sphere would have to be infinitesimal if it were only necessary that it itself should not become too large; and in order that no sphere, however small, shall become too large on being amplified by the proper proportional amount, it is necessary that the amplification of the finite sphere be indefinitely smaller still.

It might appear from this that the only absolute amplification

which can really be given to the spheres is the zero one. For the enlargement of a sphere being inversely proportional to its surface area we have the equation

$$dS/S = c/S.$$

Hence $dS = c$, a constant.

That is, the increment in the surface areas of the spheres is constant throughout the whole range, including spheres whose radii are less than any assignable quantity. But evidently, in the case of these infinitely small spheres, the increment in the surface area must be infinitely small to keep them from getting too large. Therefore the increment in the surfaces of all the spheres is infinitely small.

That this infinitely small increment is not zero, however, may be concluded, I think, from the following considerations. According to modern mathematical theory, a continuum is made up of elements in such wise that between any two of them there is a third. Now the elements of our etheric continuum are points; that is, substantive realities without dimensions. If, then, we choose any one point of this continuum, no other points exist which are in actual contact with it; just as, in the case of the proper fractions, there are no two of them which differ so little from one another that the one follows in the series immediately after the other.

A sphere, then, which contains as many as two points of the continuum contains an infinite number of them. Thus the smallest sphere of ether which we need regard as having a place in the series which was amplified above is one containing an infinite number of points, and the surface of such a sphere must contain an infinite number of points also. Since, then, the surfaces of even the smallest spheres contain an infinite number of points, and since the rate of change in the relative sizes of these infinitely small spheres is infinitely rapid, it follows that the surface increment which can be applied to any one of them without making an inner sphere equal an outer one in size is one which is itself made up of an infinite number of points. We

reach the conclusion, therefore, that the constant surface increment, which may be introduced alike at the surfaces of the infinitely great and the infinitely small spheres of ether, is an increment which is not zero, but is composed of an infinite number of the points of the continuum.

To sum up, now, the law of amplification which has resulted in the production, in our originally normal universe of ether, of the particular extensional inequality which our theory regards as the essence of an elementary unit of matter, we can say that *the relative amplifications of the spheres are inversely proportional to their surface areas, and that their absolute amplifications are such that the abnormal character introduced thereby into the general spatial relations is everywhere vanishingly slight except indefinitely close to the centre.* The essential character of this spatial inequality will now be briefly inquired into.

The volume of a sphere, expressed wholly in terms of its surface, is, according to Euclidean geometry, the product of its surface area by the length of a great circle, divided by 6π. Now consider a spherical surface having as centre the central point of the inequality. Then the volume of ether inclosed by this surface is too great — by an infinitesimal amount if the surface be finite — to be correctly expressed by that formula; and the smaller the spherical surface taken, the greater is the relative inaccuracy of the formula. This peculiar spatial state may be illustrated by the following two-dimensional analogue.

The area inclosed by a circle is equal, by the geometrical formula, to the square of the circumference of the circle, divided by 4π. If, however, we draw a circle on the surface of a sphere, the portion of the spherical surface inclosed by the circle has an area which is greater than this. Now suppose that in some way the portion of the spherical surface lying within this circle be flattened to a true plane surface, without lessening its area and without enlarging the bounding circle. Then we have a partial two-dimensional analogue of one of the three-dimensional spherical surfaces, with its excessive volume.

A quantitatively correct two-dimensional analogue, however,

would not differ appreciably from an ordinary geometrical plane, since the abnormity everywhere outside a certain infinitesimal region would be too small in degree to be detected by any physical means, and the infinitesimal region itself would be too small to be examined. Except in a strictly mathematical sense, then, the area inclosed by the circle would still be accurately expressed by the geometrical formula. But a two-dimensional analogue, to be of service as a conceptual model of the three-dimensional case, must possess more than an infinitesimal amount of abnormity, and, as I believe such an analogue may be of service in following out the further discussion of this hypothesis of matter, I will point out a somewhat closer one than that just mentioned.

If a needle-point be touched to a smooth surface of water, and then raised as far as possible without breaking its connection with the liquid, the surface of the water is drawn up from its original level; and this drawing-up takes place on all sides of the needle-point for some distance, and is especially pronounced at and close about it. Now if we imagine this cusped surface flattened down, so as to increase the diameters of all circles drawn on the surface about the cusp, but without increasing their circumferences — the area of the whole surface remaining constant during the operation — the resulting abnormally extended surface is a truer two-dimensional analogue of the form of the ether in the neighborhood of the centre of the inequality. Of course, a necessary feature of a really close analogue would be its infinite size.

II

Passing on now to a consideration of the properties of the single inequality now contained by our universe of ether, we will first show that this inequality is such that the forces of restitution, called into play by its presence in the ether, together form a perfectly balanced system, of infinite extent, about the centre of the inequality. That is, we will show that a single inequality, of the

ideally symmetrical type which we are now considering, does not alter the equilibrated condition of the ether in which it inheres.

To do this, it is necessary to define quantitatively the forces of restitution which exist in any region of the ether whose extension is not strictly normal. Now to suppose that the force at any point of an abnormal region is directly proportional in strength to the degree of abnormity at that point is the simplest assumption that can be made in this case; and it is the one which we adopt.

Such being the law which governs the strengths of the forces in the ether, it follows that it is only necessary, in order to determine what forces prevail in any particular region, to determine the degree of the abnormity which there obtains. Thus, in order to become acquainted with the system of forces which now exists in our universe, and ascertain that it is equilibrated, we have to consider the general system of the spatial abnormity.

Now, as before stated, uniform, three-dimensional amplification, when applied throughout any normal region of the ether, leaves that region still in a normal condition. In other words, the old and new ether together form a region whose extension is identical with the extension of an equally large, normal region. Of course, if we were to amplify uniformly, and by a finite amount, any finite portion of the ether, without at the same time amplifying the surrounding ether, we should get into difficulty over the fact that the bounding surface of the inner region was thereby rendered decidedly too great to make a proper junction with the inside bounding surface of the surrounding region. In our above scheme, however, no sphere was enlarged so that its surface became even equal in size to any spherical surface surrounding it. Hence this difficulty does not there arise.

Since, then, uniformity in amplification does not result in abnormity in the spatial relations, the only source of such abnormity is difference in amplification. Now in our scheme we amplified each solid sphere uniformly throughout. Hence at each step of the process we introduced abnormity only at the surface of the sphere then being operated upon. Furthermore, the degree of

the abnormity thus introduced is proportional to the amplification. For the latter is expressed by the fraction dS/S; that is, it is equal to the density of distribution of that ether which exists in the spherical surface over and above what should exist there in order that the size of that surface should be normal relatively to the sizes of the spherical surfaces surrounding it. But now the amplification was inversely proportional to the surface areas. Hence we may conclude that the forces existing at the various spherical surfaces, which may be drawn with the central point of the inequality as common centre, are inversely proportional to their respective areas; or, that the total force over any one spherical surface is precisely equal to the total force over any other spherical surface. Now since the form of the ether differs merely to an infinitesimal degree from its normal form, we may say that the forces about the centre of the inequality vary inversely as the square of the distance from that point.

It further remains to show that the force at any point of the inequality consists of two equal and opposite forces, acting respectively towards and away from the centre; that is, to show that at each point there is equilibrium.

Force exists at any point of the ether only by reason of the abnormity in the spatial relations existing between that point and the points about it. Now any point of our ether is in correct spatial relationship with the nearby points belonging to its own spherical surface, since that surface was amplified uniformly throughout. But the point is not properly related with the nearby points of the spherical surfaces which lie just outside it and just inside it; for a neighboring outside surface is relatively too small, and a neighboring inside surface is relatively too large in comparison with the surface to which the point belongs. Thus the point is acted upon by forces both from the outside and from the inside. Now these forces must be equal, since the amount of the abnormity present between the particular spherical surface to which the point belongs, and the ether to either side of it, is alike, it being true, as we have seen, that the total force over any one spherical surface equals that over any other. Moreover,

these forces from the outside and the inside are opposed to each other; for if they both were to act in the same direction, there would, since this point is not an exceptional one, be no associated reaction in the opposite direction, which would be impossible. *Hence we see that the forces acting at each point of the etheric continuum containing the inequality are in mutual equilibrium, and that therefore the whole system of forces associated with the inequality is an equilibrated one.*

The second fact regarding our gradient inequality which we wish to prove is, that in any finite region of the continuum containing the centre of the inequality, there is but a vanishingly small fraction of that whole excess of ether which was introduced in the general process of amplification.

We before saw that the increment introduced at the surfaces of the spheres, at their amplifications, was the same for the whole series. If, then, dV be the corresponding increment in the volume of a sphere of radius R, we have the relation $dV = kR$; where k is an infinitesimal constant. Integrating this expression over all space, we obtain the total volume of excess introduced.

This is, $$V = k\int_0^\infty RdR = k\,\infty.$$

Again, the amount of excess introduced at the amplifications of all spheres of radius less than x is

$$V_1 = k\int_0^x RdR = k\frac{x^2}{2}.$$

It is thus seen that the excess of ether introduced in connection with spheres of radius less than x is vanishingly small compared with that introduced in connection with all spheres of larger radius.

If, therefore, we suppose the excess introduced at each amplification simply to spread itself uniformly over the surface of the sphere taken, — as we may do, since uniform amplification results simply in a general enlargement of the sphere, — *then it follows that the whole quantity of excess in any finite region about the centre is an infinitely small part of the whole. In other words, if the excess in any such finite region could in some way be wholly removed, there*

would still be enough excess in the infinite remainder of the inequality to form an inequality, with a centre, which should differ from the original one to an infinitesimal degree only. It is true, however, that the density of the distribution of the excess increases indefinitely as we approach the centre. What accounts for the general diffused condition of the excess is the fact that the inequality is an infinitely extended one.

With regard to the absolute quantity of excess contained in the inequality, we are not in a position to tell whether it be infinitesimal, finite, or infinite. We have seen that $k\infty$, the value of the integral of dV from the centre to infinity, expresses the total quantity of excess. Here all we know regarding k is that it is an infinitesimal constant. Hence we know nothing as to the value of the whole expression. The value, however, of the integral of dV, from the centre to the surface of a sphere of radius x, is $k\,x^2/2$. Hence it follows that in the finite region surrounding the centre, the total amount of the excess is infinitesimal.

The third fact to consider pertaining to the inequality is this, that what may be termed its potential energy is so highly concentrated near the centre that all but a vanishingly small portion of it is contained by a very small sphere of ether surrounding that point.

Physical energy, in this theory, will be regarded as that condition of the ether which we have hitherto spoken of as spatial abnormity, and as nothing else. Whether the energy be so-called gravitational, kinetic, electrical, chemical, or other of the physical energies, it should be treated as essentially spatial abnormity. Thus our single inequality is in effect nothing but physical energy. The existence of the inequality depends, indeed, on the presence of an excess of ether; yet it should not be regarded as made up of that excess, for no one portion of the whole body of ether constitutes the excess rather than some other portion. Thus the inequality is essentially only of that type of spatial abnormity, or physical energy, which requires for its existence the presence of a net excess of ether. Now we have seen that the existence in the continuum of the inequality leaves unchanged the general

state of equilibrium. We may, therefore, best call the energy of the symmetrical inequality potential energy.

Having now energy to deal with, it is essential that we have a means for measuring it. The fundamental method which we shall employ in its measurement is the following: Having given a certain region of the ether, in an abnormal condition, we shall consider the total energy of that region as equal to the energy or work which would be needed in creating anew the abnormal condition there obtaining. The amount of the latter energy will be the integral of the increment of ether into the force opposing the change of spatial state.

In accordance with this scheme for measuring energy, we will now determine the potential energy of our inequality. The process by which that entity was created consisted in an infinite number of amplifications of solid spheres of ether. Each amplification gave rise to a certain amount of deformity at the surface of the sphere undergoing enlargement. Hence the amplifications involved an expenditure of work. Energy was stored up at the surfaces of the spheres. Now at each spherical surface the energy stored up was equal to the product of the quantity of ether introduced at that surface, which was dS, a constant quantity for all the spheres, by the average force existing at the surface during its introduction. Since the force is directly proportional to the abnormity, the average force was equal to one-half the maximum or final force — the final force being inversely proportional to the area of the surface. Hence the energy, dE, stored up at any surface is

$$dE = dS\frac{F}{2} = \frac{c}{R^2},$$

where c is an infinitesimal constant.

To get the total energy of the inequality we have

$$E = c\int_0^\infty \frac{dR}{R^2} = c\,\infty .$$

Again, the energy outside a sphere of radius x is

$$E_1 = c\int_x^\infty \frac{dR}{R^2} = c\frac{2}{x}.$$

Hence we have the very important result that the potential energy of the inequality is intensely localized at the centre, so much so that there is infinitely more of it within than without an exceedingly small spherical surface drawn surrounding that point.

III

THUS far we have been considering an inequality of an ideally symmetrical type, or one possessing complete internal stability, with absence of motion. We now take up a type which has the property, while preserving its form, of moving through the universe of ether in a continuous manner. That is, we are now to consider an inequality, of a special form, which has, according to this theory, the same law of right-line motion that matter has. Unfortunately anything approaching a complete or rigid discussion of this case is here out of the question.

In the first place, let us consider what it means for an inequality to change position in the general body of ether. Since the ether does not exist in space, but itself makes up space, neither the ether as a whole, nor any portion of it, ever moves in the physical sense in which material bodies do. Bodies of matter have something through which to move; viz., the ether or space. Thus bodies may exist in a state of motion, and the term inertia is applicable to them. Absolute motion and inertia, however, are terms which simply have no meaning in connection with the ether. All that can happen when the position of an inequality, or the position of its centre, changes is an alteration in the universal distribution of the spatial abnormity of the ether, or nothing but a general mutation in spatial relations. During the motion of the inequality the parts or points of the ether do, of course, change their positions relatively to one another, but their absolute velocities during the process are neither not-zero nor zero, since the term absolute motion has no meaning in connection with the parts of the self-extended continuum.

Now this peculiarity of the ether must be taken into account in considering the laws of motion of an inequality, or, in general,

the laws of propagation of an influence or wave of any kind in the ether. In an ordinary material medium the velocity of propagation of a wave is given by the expression $\sqrt{E/D}$, where E is the elasticity of the medium and D is its density. The velocity of a wave varies in this inverse manner with the density of the medium, because the propagation of a wave in such a medium involves a real movement of its parts, so that the greater the density, the longer the time consumed by the elastic forces in displacing and restoring the successive parts. With the ether, however, there is no possibility of any real motion, and therefore we may conclude that the "density" of the ether is not a factor determining the velocity of propagation of any influence or wave occurring in it.

As for the elasticity of the ether, that may be said to exist, since we have assumed that forces of restitution exist proportional to the spatial abnormity. Hence if the expression $\sqrt{E/D}$ were to indicate the velocity of ether waves, their velocity would be infinite. As a matter of fact, however, light waves are propagated in the ether at the finite velocity of 186,000 miles per second, and no physical influence is known to be transmitted at a greater velocity than that. Accordingly, it may be inferred that the expression for the velocity of waves in a material medium breaks down with the etheric continuum. What, then, does determine the speed of motions in the ether?

If a wave were to travel through the ether with an infinite velocity, it would mean that the wave, or any particular portion of it, subsisted in an infinitely great number of parts of the ether at once. But since a wave is a true entity, and has a certain amount of conserved energy, it can really subsist, at any instant of time, at only a single place in the ether. Hence results the impossibility of the propagation of an etheric wave at an infinite velocity. Furthermore, according to an assumption laid down at the beginning, one second of time corresponds with 186,000 miles of distance. Otherwise expressed, this assumption means that the infinite series of instants in one second of time is equal, as a quantitative series, to the infinite series of

points in a straight line 186,000 miles long; just as the series of points in a line one foot long is quantitatively equal to the series in another line one foot long, though unequal to the series in a line of any other length, else the two lengths would not be different from each other, but the same.

Now this assumption determines the maximum possible speed of a wave or influence in the ether. It can advance one and only one point of distance with each succeeding instant of time, and in the instants of time constituting a second can advance at most only over the points constituting 186,000 miles of distance. Whether, however, any particular influence will travel with the velocity of light, or with some less velocity, will depend on the nature of the influence. We have seen that an ideally symmetrical inequality has no tendency to move in the ether at all. What determines the velocity of an inequality whose form is different from that, or, conversely, what the form of an inequality is when its velocity has one or another definite value, is the problem which we will now take up.

An electrically charged sphere has, when removed from the neighborhood of other bodies, and at rest in the ether, a symmetrical field of radial lines of force. If, however, the sphere acquire a certain uniform, rectilinear motion, a redistribution of these lines at once takes place. The equatorial plane of the moving sphere — that is, the plane passing through the centre of the sphere and perpendicular to the line of motion — becomes a region in which the lines are crowded together, while in the forward and backward polar regions the lines become separated farther apart. This particular redistribution of the field follows as a consequence of the tendency which the lines or tubes of electric force possess of setting themselves as nearly at right angles to the direction of their motion as their own mutual repulsions will allow.

Now as an inequality consists essentially of a universal field of force and energy, we may look for a similar occurrence there. Introducing, accordingly, the geometrical construction of lines and tubes of force extending outward from the centre of the

inequality to infinite distances in all directions, we have to inquire whether these lines and tubes of force have properties similar to the electric lines and tubes, and whether, therefore, the field constituting a moving inequality is condensed to a certain extent in the equatorial plane, and correspondingly attenuated in the polar regions.

That a line of force of an inequality, or preferably a tube of small solid angle, does, like an electric tube of force, tend thus to set itself at right angles to the direction of its motion may be inferred as follows.

Any motion of the tube along its own axis tends to unbalance its forces. Thus, originally, the total normal force over any one normal cross-section of the tube equals that over any other normal cross-section. Such being the case, the ether at no portion of the tube is able to regain more nearly its normal condition at the expense of any other portion. Or, regarding the matter as a question of exchanges, as in the theory of the radiation of energy, we can say that in one second each short element of length of the tube transfers to its two neighboring elements just as much of the excess of ether as they transfer to it, and no more, the net result of the transfers being thus zero. If, however, the tube moves in the direction of its own length, the short elements form a series, one ahead of the other. Thereupon the process of exchanges is carried on more readily than before in the backward direction, and less readily in the forward direction. For suppose the velocity of the tube to be that of light. Then since no influence can travel with a greater velocity than that, there can be no passing on at all of excess in the forward direction, or no element can transmit any of its own excess to an element on ahead. On the other hand, the transfer of excess in the backward direction is greatly facilitated. Viewed from the standpoint of exchanges, therefore, there can be no suggestion of an equilbrium among the forces of a tube, moving with the full velocity of light, in the direction of its length. If the velocity be less, the gross transfer of excess in the forward direction can equal that in the backward direction only on the

condition that the total normal force over a normal cross-section of the tube increases in a continuous manner as we pass along the tube in the direction opposite to its motion. Now the general alteration in the system of forces in the tube which this new gradient distribution would involve is a process which could take place only under the action of sufficient constraining forces. It will not take place, therefore, so long as the tube can set itself at right angles to the direction of motion. If the tubes also have a tendency to keep as far separated from each other as possible, the actual motion of the tubes will be the resultant of components of motion parallel and perpendicular respectively to the axis of the tube.

This latter supposition, that the tubes tend to keep separated from each other, or in effect that they are mutually repellent, finds its warrant in the circumstance that were the tubes to come together, or mutually superpose themselves of their own accord, portions of the ether, symmetrically placed with respect to the centre of the inequality, would become, automatically, unequally abnormal in their spatial conditions. But this alteration in the distribution of the excess of ether, and of the associated forces and energy, is evidently sufficiently radical to demand for its execution the action of outside constraints.

The conception of exchanges introduced in this discussion is, very possibly, a forced one. However, if the conclusion we reach by help of it is a true one, its sufficient justification lies in its usefulness as a mechanical conception. The conclusion that we do reach is, that a measure of the velocity of an inequality is found in the ratio of the strengths of its field in the equatorial plane and along the polar axis. From analogy with the case of an electrically charged sphere, this ratio remains finite for all velocities less than the velocity of light. As that velocity is approached, the value of the ratio approaches infinity; that is, the force and energy of the inequality then become wholly imposed upon the equatorial plane. Thereupon all the forces or lines of force are at right angles to the direction of motion.

The displacement of all lines towards the equatorial plane, in

the case of an accelerated inequality, brings about important changes in its character. For one thing it tends, other things remaining the same, to increase its total energy. Thus suppose we start, as a simple illustrative case, with a symmetrical inequality, and suppose that one hemisphere of it be telescoped or superposed uniformly on the other. Then it is evident that it will, in its new condition, possess just double the energy it before had. For the forces have individually become doubled, while the total excess of ether has remained the same. Passing to the general case, it is evident that any simple displacement of the lines, which removes them, wholly or in part, from certain regions and crowds them into others, must be accompanied by an increase in the total energy of the inequality.

Now were the lines to be displaced, up to the very centre, in accordance with the law of displacement holding with electric lines of force, the total energy would be doubled long before the full velocity of light had been attained. There is good reason, however, for supposing that such an increase does not occur, especially since the energy near the centre is infinitely greater in amount than that which is associated with the inequality at a finite distance therefrom. In the case, moreover, of mutually gravitating inequalities, which we shall consider later, there seems to be no reason to believe that the amount of the combined energy of the pair suffers any change at all as the two inequalities become accelerated toward each other. It is evident, therefore, that there is some exceptional feature in the displacement of these lines.

In order to admit of a conservation of the total energy, we shall suppose that very near to the centre the lines remain undisplaced. Accordingly, whatever the velocity of the inequality, only the finite exterior portion of the total energy is doubled, or otherwise increased. The conservation of the whole is then very easily effected. What energy the surrounding portions acquire during the progress of the acceleration is drawn from the infinite reservoir of energy at the centre. The fact that this acquisition of energy by the surrounding portions, at the expense of the centre, tends to unbalance the system of forces somewhat, is not

an objection to this standpoint, since the very motion of the inequality must result from a certain unbalanced condition of that system.

We have seen that a motion of the lines of force through the ether tends to disturb their normal, symmetrical distribution. Hence to keep the distribution of lines very close to the centre undisturbed, it is necessary to suppose them at rest relatively to the ether in which they inhere. In other words, we are brought to the conclusion that at the centre the ether itself moves relatively to the surrounding ether, in the direction and with the full velocity of motion of the general inequality.

The size and shape of this moving *nucleus* are matters for speculation. The quantity of ether thus in motion is undoubtedly exceedingly small. If any definite form could be assigned to it, it might possibly be that of an elongated ellipsoid of revolution. At the same time, we must conceive the existence of such a state of continuity between the nucleus and the ether surrounding it that the condition of the one passes over into the condition of the other without any clear line of demarcation. The condition about the centre can, indeed, best be understood by examining the complex condition of the ether as a whole.

Beginning with the portions of the lines of force which are far removed from the centre, we may imagine them as displaced towards the equatorial plane by the exact amount corresponding with the speed — now supposed uniform — of the centre of the inequality. Throughout this exterior region the only motion which takes place is the uniform movement of these lines through the general body of ether. In this movement they keep up with the motion of the centre, yet really have no effective influence upon it. In other words, if the centre were to be suddenly accelerated, its motion from that moment would be practically uninfluenced by the changes which would occur with great rapidity in the distribution of these distant portions of the lines of force. This is because the important factor in the case of a moving inequality is its energy — a fact which will become evident shortly. Now the distant portions of the inequality contain

but a negligible quantity of energy, even compared with the finite amount contained by a small region which it is possible to take near the centre, and surrounding a still smaller region there in which the amount is infinite. Whence the relative unimportance of the character of the distribution of these distant portions of the lines in determining the motion of the centre.

Following the lines up closer to the centre, we come to a portion of them where the displacements help to determine the actual velocity of the inequality. The volume of this region becomes a definite one only if it be taken so arbitrarily. From the intensely localized distribution of the energy about the centre, and the minuteness of the central nucleus, it follows that we are at liberty to regard it as some very small fraction of a cubic inch. In shape it is a shell surrounding an exceedingly small central region. In this shell the displacement of the lines corresponds approximately to the velocity of the inequality. For the most part the motion is simply one of the lines through the ether. However, as we approach the inner region, a slight *drift* of the whole ether in the direction opposite to that of the motion of the inequality prevails.

Following the lines still further, we come to the minute central region whose inner portion is the moving nucleus, the notion of which we have already introduced. Analyzed more closely, the conditions in this region are as follows: —

Situated a certain distance from the outside surface of this little region — halfway in towards the centre, say — is a closed surface at which the motion is simply that of the lines of force through the ether. Outside this surface there are two motions, — a general drift of ether in the backward direction and a forward motion of the lines through this drifting ether. At a certain surface this backward drift has its maximum and grades off in both directions away from it. Inside the surface of no drift the general drift is forward. At the centre its velocity is a maximum and equals that of the inequality. As we go out from the centre, the velocity of this forward drift becomes less and less, till it reaches the value zero at the surface of no drift. As the forward

drift becomes less, the movement of the lines through the slowly drifting ether becomes correspondingly greater. Of course, in the general process, the backward drift occurs simply to offset the forward drift. Thus the amount of ether which drifts backward in any given time just equals that which drifts forward, the net drift being zero.

In the case of our uniformly moving inequality, the quantity of ether which is associated with the forward drift is, of course, perfectly definite in value. The question therefore arises as to what determines this amount. To answer this question we must take up the consideration of what is commonly called kinetic energy, or the energy associated with the motion of matter.

By the total kinetic energy of a moving inequality we herewith mean that energy which is transferred, at the time of its acceleration, from the immediate neighborhood of the centre to the general surrounding region. In other words, it is the quantity of energy possessed by the latter region over and above its normal quantity, the normal quantity being what it would possess were the whole inequality in its symmetrical state. Now there is a relation, evidently, between the quantity of kinetic energy and the quantity of ether associated with the forward drift. For example, if the latter quantity be indefinitely diminished, the former becomes indefinitely increased. For, if the velocity of the centre be fixed and finite, the increment in the energy of that portion of the inequality in which a normal displacement of the lines occurs increases indefinitely as this portion comes to include more and more of the ether immediately about the centre.

On the other hand, if the quantity of ether in the forward drift be indefinitely increased, the kinetic energy is again indefinitely increased. This follows from the fact that the continued operation of the two drifts depends on the existence of a quantity of kinetic energy specially associated with the process involved. For the backward drift is a general movement of ether relatively to the ether on either side. But there would be no tendency for such relative motion backward if the condition of the ether ahead of the centre were wholly similar to that of the ether behind it.

The actual condition ahead is necessarily one of excessive abnormity compared with the condition of the corresponding region behind. The tendency is then for the whole body of ether surrounding the nucleus to drift backward and thus destroy — if such a thing were possible — this condition of asymmetry. But the more extensive the drifts, the more extensive the asymmetry; that is, the greater the amount of the kinetic energy associated with the process. Thus the limited value of the total kinetic energy depends on the existence of a forward drift which is not too large.

In general, therefore, the kinetic energy of a moving inequality is a minimum when the forward drift has a value which is at once not so large or so small as it might be. Or again, if a certain quantity only of kinetic energy be associated with a moving inequality, its speed is a maximum when the forward drift has a certain definite size or value; viz., the value, as we shall suppose, which is assumed naturally by the inequality at the time of its acceleration.

Before deducing the law connecting the amount of the kinetic energy with the velocity, we may consider in what way, if any, this energy differs from the kind possessed by a symmetrical inequality, and called by us potential energy. Kinetic energy exists in the inequality by virtue of the fact that the symmetrical distribution of the lines of force of the motionless inequality has given place to a partly unsymmetrical distribution in which the lines appear crowded together towards the equatorial plane. The result is an increase in the energy of the displaced system of lines, as calculated by the fundamental method of energy measurement already described. The increment in the quantity of this energy is what we have termed kinetic energy. Now, evidently, this increment of energy differs in no wise in its nature from the rest of the energy which we may still regard as potential energy. That is, this increment is neither the condition of asymmetry present in the distribution of the lines of force, nor the condition of general instability which results therefrom. On the contrary, it constitutes, with the potential energy, a certain body of homo-

geneous energy. The reason for separating this body of energy into two parts, and calling one part kinetic energy, is simply the fact that an unsymmetrical distribution of the total energy gives rise to certain consequences which make it a convenience so to do. The kinetic energy of the moving inequality expresses significantly its condition with respect to general asymmetry and instability; and upon this condition depends largely its physical behavior under any given circumstances.

On the basis of the above discussion, we will now attempt to deduce the law relating kinetic energy to velocity. The displacement of a line of electric force, due to a motion of a charged sphere, is such as not to affect the straightness of the line, but only to decrease the angle which it makes with the equatorial plane. Moreover, any particular point on the line is displaced directly towards the equatorial plane, or in a direction parallel to the polar axis. Let S be the distance from the plane of any point of a line of force before displacement. Then if V be the velocity of light, and v the velocity of the charged sphere, the displacement of the point is

$$D = S\left(1 - \frac{\sqrt{V^2 - v^2}}{V}\right).$$

Differentiating this with respect to v we get

$$\frac{dD}{dv} = \frac{S}{V}\,\frac{v}{\sqrt{V^2 - v^2}}.$$

For small values of v we have, approximately,

$$\frac{dD}{dv} = \frac{S}{V^2}v.$$

Integrating this we get

$$D = \frac{S}{2V^2}\,v^2,$$

which holds for all values of v which are small in comparison with V.

With this law of displacement holding true also for the lines of force of our inequality, we can proceed to show that its kinetic energy varies as the square of its velocity. Take any small

cubical portion of the field of a symmetrical inequality. Then as the inequality is set in motion — the direction of motion being parallel to four of the edges of the cube — this whole cubical part of the field is displaced directly towards the equatorial plane. In the process its height is shortened, the reduction being proportional to v^2. This follows simply from the last equation deduced. Thus the decrement in the volume of the original cube varies as the square of the velocity. Accordingly, if the field of the symmetrical inequality be supposed completely divided up into little cubes, it is seen that the individual volume of every small portion of the field becomes reduced by an amount which varies as the square of the velocity. But this means that the individual energy of each of these small portions, and thus also their combined energy, is increased by an amount proportional to v^2. Thus we have the result that the kinetic energy of a moving inequality, so far as it comes from the natural displacement of the lines of force, varies directly as the square of the velocity.

Another increment in the energy results from the unsymmetrical condition directly associated with the forward and backward drifts. In this unsymmetrical condition the abnormity is greatest ahead of the centre and least behind. That is, there is an abnormity gradient from front to back, and its steepness varies directly as the velocity of the centre, since doubling the velocity doubles the quantity of ether which the drifts have to transport in a given time. But now in the establishment of the gradient the increment of energy varies as the square of its steepness. The principle involved here is the same as that which determines that the energy of twist in a wire shall increase as the square of the angle of twist. In both cases the forces opposing the establishment of the new state increase as the square of the variable. In the one case the variable is the angle of twist, in the other the quantity of ether transferred from back to front in the establishment of the gradient. *Since, then, this second increment varies as* v^2, *as did the first, we may state the law that the kinetic energy of an inequality varies as the square of its velocity.*

In concluding our discussion of simple motion, it remains to

indicate just why an inequality, possessing a definite amount of kinetic energy — or existing with its lines under definite displacement, and being provided with a gradient of abnormity about its nucleus — should have the property of continuous, uniform, right-line motion.

Our explanation is based on the following postulate regarding the behavior of the general body of ether. Through every general change in the condition of the etheric continuum which can cccur of itself, the essential character of that condition remains permanently the same. In other words, we postulate that any one general state of the ether must of necessity be equivalent, if rightly viewed, to any possible preceding or succeeding general state. This means that a single, unsymmetrical (non-radiant) inequality, if left to itself, can never by any possibility become a symmetrical one, or an inequality possessing either more or less essential asymmetry or instability than it has to start with.

Now if the motion of our inequality be continuous, uniform, and right-line, its own condition will naturally be completely conserved. The only change then occurring in the general body of ether is a continuous shifting of the distribution of excess, as a whole, in a definite direction. The result may thus fairly be considered a continuous preservation of the essential spatial character of the etheric continuum. It is, moreover, no longer possible, as in the case of a perfectly symmetrical inequality, for the ether to preserve its extensional state through an unvarying distribution of the excess about a fixed point. *Thus the sufficient reason for the continuous, uniform, and right-line motion of the inequality is the fact that such motion is the most natural means whereby the essential condition of the general body of ether can be continuously conserved.*

IV

We now take up the important subject of the mutual gravitation of two or more inequalities. In thus introducing a plurality of these elementary units of matter, we make the assumption

that they are all similar one to another; that is to say, they result from equal quantities of the excess of ether.

Beginning with a single pair of inequalities, it is evident that if their centres be separated by so vast a distance that each virtually exists in a universe by itself, it is then possible to regard both as perfectly symmetrical and undisturbed. On the other hand, if the distance between centres be finite, the two are co-extensive, and a mutual action of some kind may be looked for. In order to determine the nature of this action, we have first to learn the normal state of the universe of ether corresponding with the co-existence in it of two inequalities with centres a finite distance apart.

To begin with, we may define what is to be meant by the simple superposition of one inequality upon another. Let a single symmetrical inequality subsist in the ether. Then suppose a second inequality to be created, about another centre, by the process we formerly employed in the production of a first one. The result is the simple superposition, corresponding with the given positions of the centres, of the second inequality upon the first, or vice versa. The original process, it is to be observed, is still productive of the same general change in spatial state as formerly. For the presence of the first inequality has altered the general normal character of the ether to an infinitesimal degree only. Hence the effective spatial conditions under which the second increment of volume is introduced are virtually the same as existed at the introduction of the equal, first increment; and, therefore, just the same infinitesimal alteration in spatial state must result.

To obtain the force at any point after the simple superposition, we proceed as follows. Draw the two vectors, representing respectively, both in magnitude and direction, the two forces which would exist successively at that point if first one and then the other inequality existed separately about its centre. Find the resultant of these two vectors. This resultant represents, in magnitude and direction, the force at the point.

This result is, of course, what would be anticipated from the

ordinary theory of the composition of forces. We shall, however, not be satisfied with this justification of our result. Let us see if we are not able to establish it on the basis of our own conception of the spatial origin of force.

We have seen that normal force exists at a surface drawn in the ether when its area is too small in comparison with the parallel surfaces on one side of it, and too great in comparison with the parallel surfaces on the other side. The surfaces which we previously considered were spherical, with common centre at the centre of the inequality. Confining our attention to any very small region of the ether, away from the centre, the portions of the spherical surfaces which lie within this region have little curvature and are nearly planes. Moreover, the field of force in this region may be regarded as uniform. Call its value f. Then f represents the integral of the normal force, over unit surface, at right angles to a radius of the inequality passing through a point of the small region. If we take any plane at right angles to this first plane, the normal force over it is evidently zero, since the parallel planes on either side of it have the same areas as itself. Now what is the normal force over a plane making any angle θ with the first plane?

Assuming the formula for the force to be $f \cos \theta$, its value reduces, as it should, to f and 0 respectively when θ is 0° and 90°. Moreover, it is the true one, as can be seen in the following manner. Imagine a magnifying glass such that, when rightly oriented and used in observing a very small square of paper, two of the opposite sides, say vertical sides, appear as slightly diverging straight lines, while the horizontal sides still appear parallel. Then if two lines be drawn on the square, close together and parallel to each other and to the vertical sides, they will appear, when looked at through the glass, to diverge by a certain small angle ϕ. Now draw two other parallel lines on the square, the same short distance apart, and making an angle θ with the first pair. Then, as a simple geometrical construction will show, this latter pair will appear to diverge by an angle $\phi \cos \theta$. Substituting equal real amplification for this magnifi-

cation, and regarding the paper as having the properties of the ether, the actual angles of divergence of the two pairs of lines then measure the strengths of the forces in their respective directions. Hence the cosine law holds true. In other words, the force at a point in a direction making an angle θ with the radius is $f \cos \theta$, where f is the force along the radius.

Returning now to the case of the pair of inequalities, the resultant force at a point is thus

$$R = f_1 \cos \theta_1 + f_2 \cos \theta_2,$$

where f_1 and f_2 are the forces in the directions of the radii due to the inequalities singly, and θ_1 and θ_2 are the angles which the resultant makes with the two radii. R is evidently a maximum for some direction lying in the plane determined by the two radii. Thus in getting this maximum we may write

$$R = f_1 \cos \theta_1 + f_2 \cos (a - \theta_1),$$

where a is the angle between the two radii.

Differentiating,

$$\frac{dR}{d\theta_1} = - f_1 \sin \theta_1 + f_2 \sin (a - \theta_1).$$

For the maximum,

$$f_1 \sin \theta_1 = f_2 \sin (a - \theta_1).$$

Or

$$\frac{\sin \theta_1}{\sin (a - \theta_1)} = \frac{f_2}{f_1}.$$

That is, the maximum resultant is represented, both in magnitude and direction, by the resultant in the ordinary triangle or parallelogram of forces.

Now in the equation

$$R = f_1 \cos \theta_1 + f_2 \cos (a - \theta_1)$$

the second member represents a simple cosine function of θ_1. Hence R is symmetrical with respect to its maximum value. In other words, the forces at a point, in directions other than that of the maximum resultant, so pair off with each other that the total resultant is in the same direction as the maximum resultant, while

the force at right angles to it is zero. Thus the force at any point in the ether, due to a pair of simply superposed inequalities, is wholly similar in character to that set up at a point by one alone.

Our general result thus far, then, is this, that the system of forces of two simply superposed inequalities is exactly similar to the gravitational field about two equal material particles; provided we make allowance for the fact that these latter centres of force are not points, and recognize that on that account the analogy breaks down as we approach the point-centres of the inequalities.

Now it is a well-known property of this system of forces that the integral of normal force over any one normal cross-section of a tube equals that over any other, and that this property of equilibration guarantees to this type of system a permanent place in the economy of the etheric continuum may be seen as follows. Starting with a pair of simply superposed inequalities, with centres a certain finite distance apart, suppose a gradual, but in the end infinite, separation of the two centres to take place. And suppose the manner of separation to be such that, whatever the positions of the centres may be at any moment, the two inequalities are then simply superposed. Then when the distance between centres becomes infinite, the inequalities attain to their symmetrical forms and are independent of each other.

Reversing the process, but allowing the universe of ether to adjust itself automatically to the continuous change in position of the centres, through what series of conditions will this universe now pass? Evidently the series will have to be one in which each term corresponds with a system of forces which is at least approximately equilibrated. For the series begins with a perfectly equilibrated system. Now if a markedly unbalanced system of forces were to appear somewhere in the series, it would mean that some portion of the ether — viz., that portion where the forces overbalanced the opposing forces — had failed to regain more nearly its normal condition when it was possible for it so to do. But this state of affairs is impossible. Hence the system of forces passes continuously from one (approximately) equilibrated

form to another, thus exactly reversing the process which took place upon the separation of the centres, and eventually restoring the continuum to the original condition.

Thus this condition of simple superposition is a characteristic one for the ether to assume. It admits, in all cases, of a conservation of the total excess of ether; and while it would, if mathematically normal, increase by a finite amount the total energy of the pair of inequalities, an infinitesimal change of condition about the two centres counteracts this tendency away from conservation. In general, since it is necessary for the forces in the ether to approximate very closely to general equilibration, we may regard this as the typical condition when only two inequalities are involved. With more than two inequalities we should have as the typical condition the one corresponding with the simple superposition of all the inequalities which there happen to be.

Up to this point we have been discussing the preliminary problem of the characteristic state of the ether when there exists more than a single inequality, and in particular when two exist. We now inquire concerning the consequent action of one inequality upon a second.

Fixing our attention on the condition of the ether in the neighborhood of one of the centres — the other centre being removed a finite distance — it appears at once that the ether on the side away from the second centre is in a less normal condition than the ether on the side toward that centre. For the strength of the forces measures the abnormal character of the ether, and on the former side the force which naturally exists there, due to the second inequality alone, is added to the forces of the first inequality; and on the latter side subtracted. Thus in passing from the further side to the near side of the first inequality, the degree of the abnormity drops by an amount equal to twice the strength of the force at that point due to the second inequality alone.

Now if the centre were to move in the direction of the second centre, the ether on the further side would regain more nearly its normal condition, and the ether on the near side would be

forced further from the normal condition. Consequently this motion towards the second centre will actually take place, since the further side, having greater abnormity, is in a position to regain more nearly its normal form at the expense of the near side, with its lower degree of abnormity or weaker forces. A tendency therefore exists for the centre of the inequality to move towards the second centre, and this tendency is proportional in strength to twice the force at the centre due to the second inequality singly. In other words, the first inequality is attracted towards the second with a force which is inversely proportional to the square of the distance between the centres. Similarly the second inequality is attracted towards the first according to the same law.

This definite force of attraction exerted upon an inequality, in consequence of the greater or less nearness of a second one, results in the accelerated movement of each centre towards the other. The acceleration at any point is such that the whole attractive force is employed in producing change of motion. In terms of energy, this means that the space rate of increase in the kinetic energy of each of the mutually gravitating inequalities is proportional at each point to the attractive force upon it. Now the law of kinetic energy and the laws of motion for inequalities are those holding for actual matter. Thus the tendency to motion of two mutually attracting inequalities is entirely analogous to that of two equal material particles. *Whence we have the result that the gravitational behavior of a pair of inequalities is just what it should be on the hypothesis that they are equal elementary units of matter.*

This principle of the mutual action of two inequalities may now readily be expanded into a law of the *universal gravitation of inequalities. For since, as we saw, any number of inequalities, having their centres disposed in any manner, may be simply superposed upon each other, the result of the coexistence of any number of them will be that each inequality attracts, and is attracted by, every other inequality, with forces varying inversely as the squares of the various distances between centres.*

In concluding our discussssion of the mutual gravitation of in-

equalities, let us consider what happens when the centres of two of them come very close together. On approaching coincidence, the velocities naturally become very great, approximating to the velocity of light. By reason of this great velocity, and the alteration in their forms which thereby results, the force of their mutual attraction becomes small. Their motions, however, still bring the centres nearer together. In consequence, the two moving nuclei tend to oppose and stop each other. At the same time, the lines of force of the one nucleus tend to superpose themselves upon the lines in the other nucleus. But a complete superposition is impossible, since that would necessitate a doubling of the total energy of the system. After a certain amount of superposition has taken place, therefore, a reaction occurs between the central regions. The two inequalities repel each other, and a complete reversal of their former motions results. Since, however, the nuclei are most likely of an elongated form, a complete reversal of this sort will occur but rarely. In general, when the two nuclei do not pass by each other in nearly straight lines, they rebound at one or another angle.

V

Thus far we have been considering matter only in the form of its primary units, the simple inequalities. It remains to consider it briefly in the form of atoms, molecules, and bodies.

While inequalities are, from some constitutional or else evolutionary cause, all of the same degree or mass, atoms have various masses. The atom of smallest mass is the hydrogen atom. According to the electron theory of matter, such an atom is built up from about a thousand electrons or primary units of matter. According to our present theory we may suppose it built up of about *a thousand inequalities*. Heavier atoms are proportionally larger collections of them.

In any atom the units are held together by their gravitational attractions. The reason why there is but a limited number of

different existent types of atomic systems is because only those special types are peculiarly stable or well-balanced.

Atoms in combinations form molecules, and molecules similarly in combinations make up sensible bodies. Thus we have the complete synthesis of the material universe from the elementary, equal inequalities.

In general, these large aggregates borrow from their component inequalities the fundamental properties which we have found the latter to possess. Thus the laws of motion, of kinetic energy, and of gravitation apply to atoms, molecules, and sensible bodies as well as to simple inequalities. On the other hand, these aggregates naturally possess many unique properties which the individual inequalities do not; or again, if not unique, properties which they hold at least in their own right. Thus the attribute of repulsion, as manifested when any two such aggregates collide, seems to be a direct one of the aggregates as such, and referable, if at all, only indirectly to the like attribute of the distinct inequalities. Upon coming together, the presence of each system disturbs the normal organization of the other, with the result that the aggregates are driven apart in a general redistribution of the excess of ether.

This concludes the part of our theory which relates purely to the unelectrified or normal form of the physical entity matter. A brief hypothesis will be appended as to the nature of electricity or electrification and the conditions underlying a few electrical phenomena.

VI

The process of electrification involves as its principal characteristic, as we shall suppose, a disjunction of one or more inequalities into two parts. Calling to mind the structure of a moving inequality, it is seen that the most natural single division of it leaves the forward-moving nucleus and the adjacent backward drift united in one part, and includes in the second part the whole surround-

ing portion, or that portion where the lines of force themselves move through the continuum. The forward and backward drifts are, of course, inseparable, since the one is required to neutralize directly the general effect of the other.

To bring about this disjunction it is necessary that the two parts of the inequality be acted upon by other inequalities. As a rule, we may regard any inequality undergoing disjunction as belonging to an atom which is in close contact with a second atom. In the simplest case only one inequality of the first atom is broken up. If we denote the principal extensive part resulting from the disjunction as the *field* of the inequality, we may then say that the first atom retains control of the field of its own disjunct inequality, while the second atom gains control of the *nucleus*. By "control" is here meant a preponderating gravitative force or attraction. Along with the forward-moving nucleus, the second atom takes over the backward drift of the disjoined inequality; but since this follows as a matter of course, we shall, in general, speak only of the whereabouts of the nucleus, it being understood that immediately surrounding that is always the backward drift.

At the moment, then, when electrification is being established, one atom has control, as before, of the field of a certain one of its inequalities, while a second atom is gaining full control of the nucleus. The agency which pulls an inequality apart is thus a pair of atoms in the act of rapid separation immediately following a collision or other intimate contact. In the sudden processes then occurring, the nucleus severs its normal connection with its own field. An adjacent inequality of the second atom strongly attracts and holds it to itself. Instead, therefore, of regaining its normal position relatively to its own field, the nucleus is carried away by the motion of the entire second atom. The result is an electrification of the two atoms. The atom retaining the field is *positively* electrified, while the atom gaining possession of the one nucleus in excess is *negatively* electrified.

In this account we suppose that an atom, or one of its component inequalities, is able to attract the nucleus of an inequality

when removed from its field, and likewise is able, by a similar attractive force, to maintain possession of a separated field. That such is the case becomes evident at once when, in place of one of the inequalities of a gravitating pair, we substitute, first only its nucleus, and then only its field. In the one case the gravitation of the nucleus, and in the other case the gravitation of the more central portion of the field, towards the centre of the complete inequality, occurs for essentially the same reason that the gravitation of the centre of a second complete inequality itself would.

We do not suppose, however, that in this case of two oppositely electrified atoms we are dealing purely and simply with a formal displacement of a nucleus relatively to a field. If that were so, we should have no such thing as an electric field about the oppositely charged atoms, or an electric attraction between them. The real complexity of the case is much greater than that.

Now the separation of the nucleus from the field is necessarily a forced process, since it involves the setting up of an unsymmetrical and disequilibrated condition in the ether. Supposing the separation to proceed, however, in spite of the associated reaction, there goes on simultaneously a certain partially compensating readjustment. *This is the electric polarization of the medium.* It consists in a separation of the ether into a series of layers such that any two adjacent ones have opposite relative states, the one layer containing *too much* ether relatively to the other, and the other thus containing relatively *too little.* The surfaces of these layers are supposed to be the equipotential surfaces about the nucleus and central portion of the field, considered as two equal and opposite charges. The thickness of a layer at any part of the field is inversely proportional to the electric force there. Thus if we draw in the series of equipotential surfaces corresponding to equal differences of potential, and make the common difference small enough, we then have the whole field mapped out into a series of regions which coincide with the series of layers constituting the electric polarization.

To bring the ether of this series of regions into its proper polarized condition, it is necessary to transfer quantities of ether from alternate regions of the series to the regions which lie in between them. Or rather, what really takes place is a general systematic alteration in the spatial relations of the ether of each successive pair of regions whereby the one region is left with too much ether relatively to the other. The layer of each pair having the over-supply of ether is the one farther removed from the nucleus or negatively electrified atom. Thus the transfer of ether in the field which establishes the polarized condition takes place *toward* the positive charge, or along the lines of force and in their negative directions.

Just to illustrate how a polarized condition of the ether of the general type above suggested might be established, consider the following. Suppose we have the nucleus separated indefinitely far from the central portion of the field. Then the equipotential surfaces about the nucleus are spheres, with the nucleus as common centre. Take three of these surfaces, at some distance from the nucleus. They are the boundaries of two, adjacent, thin spherical shells. We now wish to bring the ether of these two shells to the polarized condition.

To simplify the case, we shall suppose that the ether of the two shells is normal to start with, or that the existence of the disjunct inequality as such, or the existence of any other inequality, does not affect that region of ether. Then the transfer of a certain quantity of ether, from the smaller shell to the larger one, may be effected as follows. Start with the outside equipotential surface, and let the solid sphere of ether bounded by that surface be amplified positively by a certain amount, so that the surface becomes too large relatively to the surfaces surrounding it. Continue the process with the smaller spheres, till a sphere equal in size to the middle equipotential surface is reached. This will happen, as is evident, somewhat less soon because of the positive amplifications. Then starting with the solid sphere bounded by that surface, let it and the smaller spheres be amplified negatively — if that expression may be allowed — till the

inner equipotential surface be reached. This latter surface is reached somewhat sooner because of the negative amplifications. Then the result is a simple transfer of ether from the inner shell to the outer one, provided only that the series of positive and negative amplifications be such that the same amount of ether is added through the former as is subtracted through the latter. Carrying on the same operation with every pair of shells, as marked out by the series of equipotential surfaces, we then have the whole field about the nucleus reduced to a certain polarized condition.

In an electric field, the energy per unit volume varies from region to region directly as the square of the electric force, and in the case of an isolated charge the latter varies inversely as the square of the distance. Now it is seen that the above process of polarization introduces energy throughout the region surrounding the nucleus, and it is quite possible to give a simple rule of polarization such that the above law of the distribution of electric energy is the one which holds. For example, it is only necessary to proceed with each pair of shells as follows: First, make the positive amplifications start with the value zero at the outer equipotential surface, increase in a linear manner to a maximum value at the middle of the outer shell, — the value of the maximum being made proportional to the electric force there, — and decrease similarly to zero at the middle equipotential surface; and secondly make the negative amplifications follow the same law down to the inner equipotential surface. However, since the particular type of polarization thereby obtained would very possibly differ in an essential manner from the true type of electric polarization, we shall not stop to carry out the proof.

It seems necessary, indeed, to suppose that electric polarization is a varying rather than a fixed condition of the ether. For any polarized state is evidently not one in which the ether is equilibrated, as it is in the case of a single symmetrical inequality. The actual condition may thus be supposed to vary rapidly in a cyclical manner, any one particular condition of polarization recurring again after some short interval. Throughout the cycle

of polarized states the electric energy per unit volume remains constant, any one polarized condition in the cycle being essentially equivalent to any other. As for the general nature of the cycle, we may suppose it to be oscillatory rather than rotary. That is, the general cyclical changes occurring in the character of the spatial relations are such as could be brought about by a transfer of ether backwards and forwards parallel to the lines of electric force, rather than by a transfer in circular or elliptical paths.

In this discussion of the nature of electric polarization in the simple ether, we have not been able to describe definitely either the character of a particular state of polarization or the cyclical process by which one state passes over into another possessing the same amount of energy. A circumstance which renders exceptionally difficult the analysis of the states and processes occurring in an electric field — as well as in a magnetic field, or in electromagnetic waves — is the possession by our universe of ether of an extreme degree of plasticity. This plastic character is due to the mutability of the spatial relations, and is readily appreciated immediately one attempts to determine the probable behavior of an abnormal region of ether, even in some very simple cases. For instance, if a certain etheric shell contains too much ether relatively to another one which immediately surrounds it, the spatial abnormity obtaining between the two may be corrected in a great variety of ways: viz., (1) by a transfer of ether from the first shell directly to the second; (2) by a transfer of ether from the first shell to the region inside it; (3) by a transfer of ether to the second shell from the surrounding ether; (4) by one or another combination from these methods. In any event, the correction is made, if at all, in a subtilely direct manner, through a general alteration in a large system of spatial relations. Such being the case, the determination of the exact nature of electric polarization stands as a serious problem.

With regard to the electric attractions and repulsions produced through electric polarization, we may suppose them due in all cases to a greater concentration of the polarized condition on one side of the charged body than on the other, the body always

tending to move in the direction of the stronger polarization. The force acting on the charge is measured by the time rate of increase in the total energy of polarization when the charge is moving with uniform, unit velocity against the force.

In order to account for the fact that the electric attraction between the parts of a disjoined inequality is vastly greater than the gravitational attraction between two normal inequalities, we may suppose either that the forces in the electric field are as a whole vastly greater than those in the gravitational field, or else that the effective difference in the strength of the forces of polarization on opposite sides of the nucleus — as well as on opposite sides of the most central portion of the separated field — is alone vastly greater than the similar difference in the case of the gravitating inequalities. The latter supposition appears to be the more plausible one, and the strong resultant force upon each centre of polarization which this view calls for probably exists as a consequence of the general coarseness of structure of that polarization.

VII

I SHALL close this paper with a few short suggestions (for the most part borrowed from existing physical theories) as to the further nature of electric, magnetic, and electromagnetic phenomena.

An electric current consists in the handing on, from atom to atom and molecule to molecule, of nuclei or elementary negative charges. In a wire carrying a current there is, however, no general excess of these nuclei due to the presence of the current, since the moving nuclei are those which naturally belong with the atoms of the wire itself.

The mechanism which drives an electric current is a magnetic field which at once surrounds and permeates the conductor. The

energy dissipated as heat in the conductor reaches its surface as magnetic energy of the surrounding dielectric.

A magnetic field differs from an electric field in that the cyclical change in it is rotary in character, not plain oscillatory. That is, the general cyclical changes occurring in the character of the spatial relations are such as could be brought about by a transfer of ether in circular or other area-enclosing paths. An electric field becomes partly magnetic in character when the lines of force are given a motion at right angles to their length.

A "line" of magnetic force about a conductor carrying a current is a sort of etheric vortex-ring. Its direction of rotation is such that the motion on its interior side is in the direction in which the nuclei are moving in the conductor, or that direction along the conductor in which the potential increases.

A permanent magnet consists of a large number of continuously flowing, closed molecular currents oriented to a greater or less extent in one way. A paramagnetic substance is one whose molecules are provided naturally with such currents; which currents, under the action of a magnetic field, become similarly oriented. A diamagnetic substance, on the other hand, is one whose molecules do not possess continously flowing currents, but in which molecular currents may be induced by the action of a magnetic field.

An electromagnetic train of waves consists of a series of polarizations, alternately electric and magnetic, in process of propagation through the ether. The direction of propagation is perpendicular both to the electric and magnetic forces, and these forces are themselves at right angles to each other. An etheric disturbance of this nature, and of short wave-length, constitutes light.

It happens that, under certain conditions, the separated nuclei of inequalities are ejected with violence from the atoms and molecules with which they naturally associate themselves. Thereupon they exist, for a time, *as free nuclei*, and constitute either the negative electrons or their counterparts the so-called β rays from radio-active substances.

The positive electrons and a rays are, on the contrary, the separated fields of inequalities *still in association with atoms of matter.* This linkage of the elementary positive charges with the relatively heavy atoms is one which it is impossible, so far as experiment has yet shown, to break.